ΔK = 0 M1 Excitation Strength of the Well-Deformed Nucleus ^{164}Dy from K Mixing

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The size of a $\Delta K = 0$ $M1$ excitation strength has been determined for the first time in a predominantly axially deformed even-even nucleus. It has been obtained from the observation of a rare K-mixing situation between two close-lying $J^\pi = 1^+$ states of the nucleus ^{164}Dy with components characterized by intrinsic projection quantum numbers $K = 0$ and $K = 1$. Nuclear resonance fluorescence induced by quasi-monochromatic linearly polarized γ-ray beams provided evidence for K mixing of the 1^+ states at 3159.1(3) and 3173.6(3) keV in excitation energy from their γ-decay branching ratios into the ground-state band. The $\Delta K = 0$ transition strength of $B(M1; 0_1^+ \to 1_{K=0}^+) = 0.008(1)\mu_N^2$ was inferred from a mixing analysis of their $M1$ transition rates into the ground-state band. It is in agreement with predictions from the quasiparticle phonon nuclear model. This determination represents first experimental information on the $M1$ excitation strength of a nuclear quantum state with a negative $\mathcal{R}$-symmetry quantum number.

Introduction.—Symmetry properties play a paramount role in the description of physical systems. In classical mechanics, this is highlighted by Noether's theorem [1] stating that every symmetry of the Hamiltonian function is related to a constant of motion. In particular, in the modeling of fundamental forces and quantum-mechanical systems, e.g., atomic nuclei, symmetry properties are exploited. Examples from nuclear physics are chiral effective field theory [2], the interacting boson model [3], or the collective model of Bohr and Mottelson [4]. For instance, these enable the description of nuclei which take non-spherical equilibrium shapes potentially breaking rotational symmetries. An invariance of the deformation with respect to a subgroup of these symmetries results in a restricted rotational degree of freedom. Inversely, the occurrence of additional rotational degrees of freedom is rooted in the symmetry breaking of the underlying system. Axially symmetric nuclei, for instance, exhibit rotational degrees of freedom perpendicular to the symmetry axis, while the

rotation about it remains part of the intrinsic degrees of freedom. The symmetry of the deformation then restricts the quantum numbers of the respective rotational spectra. Examples of such symmetries (quantum numbers) are space-reflection invariance (parity $\mathcal{P}$, $\pi = \pm 1$) and the invariance with respect to a rotation of 180° around an arbitrarily chosen axis perpendicular to the symmetry axis ($\mathcal{R}$ symmetry, $r = \pm 1$) [4] as depicted in Fig. 1. A rotational band with $K^\pi = 0^+$, where K is the projection of the total angular momentum quantum number onto the intrinsic symmetry axis, comprises only states with even (odd) angular momentum J for $r = +1$ ($r = -1$):

$$J = \begin{cases} 0^+, 2^+, 4^+, \dots & \text{for } \pi = +1 \wedge r = +1 \quad \text{and} \\ 1^+, 3^+, 5^+, \dots & \text{for } \pi = +1 \wedge r = -1. \end{cases} \quad (1)$$

In the case of $K > 0$, the well-known selection rule $J = K, K + 1, K + 2, \dots$ holds. As the possible angular momenta fulfill the condition $(-1)^J = r$ for a $K = 0$
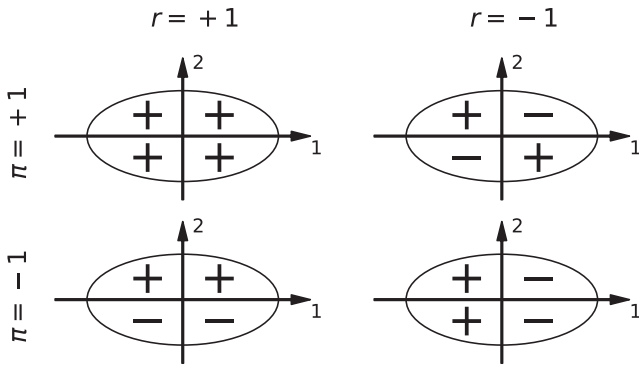
FIG. 1. Schematic representation of the interplay of $\mathcal{P}$ symmetry and $\mathcal{R}$ symmetry. The symmetry axis of the nuclei in the body-fixed coordinate frame is denoted by 1 and a generic axis perpendicular to the symmetry axis by 2. The signs denote the spatial distribution of the phases of the collective wave function over an axially deformed nucleus. With respect to a rotation of 180° about 2, the nuclei on the left-hand side are symmetric. Thus, they are $\mathcal{R}$ invariant with positive eigenvalue r whereas the nuclei on the right-hand side carry $r = -1$.

rotational band, $\mathcal{R}$ symmetry can be assigned unambiguously. While the sequence of angular momenta for $r = +1$ corresponds to the well-known ground-state rotational bands of axially symmetric deformed nuclei, information on potential $J_K^\pi = 1_{K=0}^+$ states is still missing. From the analysis of photon-scattering data it has been assumed that such $1_{K=0}^+$ states are only weakly excited below 4 MeV [5]. In this energy region, the largest fraction of the magnetic dipole ($M1$) excitation strength is claimed by the $K = 1$ scissors mode [6,7].

It is the purpose of this Letter to present the first information on the $M1$ excitation strength and decay characteristics of a $J_K^\pi = 1_{K=0}^+$ state. This information has been extracted by precisely measuring branching ratios between excited 1^+ states around 3 MeV and the ground-state rotational band of the well-deformed nucleus ^{164}Dy. In parts, these branching ratios deviate substantially from Alaga-rule [8] predictions. These differences, for states with angular momentum quantum number $J = 1$, can only occur by mixing of underlying basis states with projection quantum numbers $K = 0$ and $K = 1$. Experimental information on $M1$ excitations with projection $K = 0$ is missing so far for axially deformed even-even nuclei. The studied K-mixing scenario allows for the extraction of the corresponding K-mixing matrix element and the $\Delta K = 0$ $M1$ transition strength for the first time. Moreover, this represents the first finding of a member of a positive-parity rotational band with a negative $\mathcal{R}$-symmetry quantum number.

Experiment and results.—Nuclear resonance fluorescence experiments [9–11] have been performed at the High-Intensity γ-Ray Source (HIγS) [12] at Duke University, Durham, North Carolina. The incident linearly polarized quasimonoenergetic γ-ray beams [13,14] were

scattered from a target composed of 0.770 g Dy$_2$O$_3$ and 1.1 g metallic dysprosium with enrichments in ^{164}Dy of approximately 98% and 95.6%, respectively. The target was mounted in the γ^3 setup [15] consisting of four high-purity germanium (HPGe) detectors at positions (ϑ, φ) of (90°, 0°), (90°, 90°), (135°, 315°), and (135°, 225°) alongside four large-volume, cerium-doped lanthanum bromide (LaBr$_3$:Ce) scintillators for $\gamma\gamma$-coincidence measurements. The latter were placed at positions (90°, 180°), (90°, 270°), (135°, 45°), and (135°, 135°). Here, polar (azimuthal) angles ϑ (φ) are defined with respect to the outgoing beam (the beam's horizontal polarization plane). In order to reinvestigate the decay characteristics of known 1_i^+ states ($i = 1$–3) at 3111.2(3), 3159.1(3), and 3173.6(3) keV [16–18] and probe the existence of a nuclear state at 3100 keV which was tentatively conjectured previously [18,19], measurements with beam energies of 3075(50) and 3180(52) keV were performed. In the first setting, the 1_3^+ state at 3173.6(3) keV is excluded from the excitation region due to the beam's small energy spread. Thus, the ground-state decays of potential state(s) at 3100 keV are not masked by the $1_3^+ \rightarrow 2_1^+$ transition at virtually the same energy. The corresponding $(\vec{\gamma}, \gamma')$ spectra are shown in panel (a) of Fig. 2 for a detector in the polarization plane (solid blue) and perpendicular to it (dotted red). In the spectra corresponding to the second beam setting [cf. panel (c) of Fig. 2], transitions of the 1_i^+ states ($i = 1$–3) to the 0_1^+ ground state and the 2_1^+ state at 73.393(5) keV are observed prominently. Panels (b) and (d) of Fig. 2 display $\gamma\gamma$-coincidence spectra requiring a coincidence with the γ ray from the $2_\gamma^+ \rightarrow 2_1^+$ transition at 688.42(1) keV recorded in the LaBr$_3$:Ce scintillators. This measurement provides an unambiguous test for the proposed coupling of the states at 3100 and 3173.6(3) keV to the 2_γ^+ state [18,19]. Indeed, these transitions are identified in the coincidence spectra at 2338 and 2411 keV, respectively. Consequently, experimental decay-intensity ratios $I_{1 \rightarrow f/0}^{\text{rel}} = \Gamma_f / \Gamma_0$ have been determined for the 1_i^+ states ($i = 1$–3). Here, Γ_f denotes the partial decay width to an excited state (2_1^+ or 2_γ^+) and Γ_0 to the 0_1^+ state. The spectra recorded at a beam energy of 3075(50) keV along with the $\gamma\gamma$-coincidence measurement prove the existence of $J = 1$ states of ^{164}Dy at 3100 keV. The quantitative analysis is consistent with a doublet of a $J^\pi = 1^+$ and a 1^- state featuring γ decays to the 0_1^+, 2_1^+, and 2_γ^+ states with transition energies of 3100, 3027, and 2338 keV, respectively. This aspect will be discussed in a forthcoming publication. In panel (a) of Fig. 4, a revised level scheme is shown, which includes the states at 3100 keV and their decay channels. A coupling to the $K = 2$ γ-vibrational band is also observed for the 1_3^+ state at 3173.6(3) keV. It is emphasized in the following discussion that it can be shown that the γ-ray peaks associated with the decay transitions of the 1_3^+ state contain neither
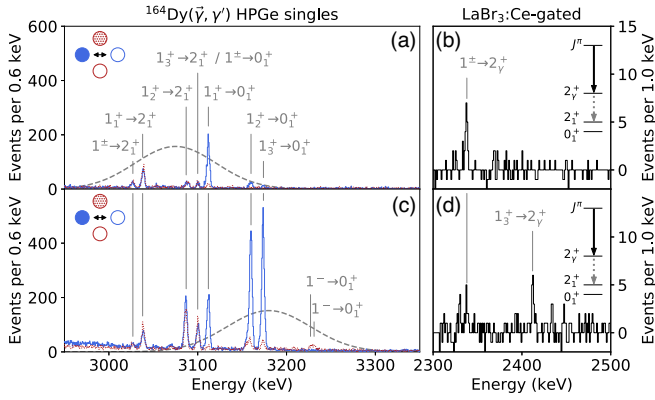
FIG. 2. γ-ray spectra from the ^{164}Dy$(\vec{\gamma}, \gamma')$ reaction taken at the HIγS facility [12] with beam energies $E_{\text{beam}} = 3075(50)$ keV [panels (a) and (b)] and $E_{\text{beam}} = 3180(52)$ keV [panels (c) and (d)]. Detectors were placed at a polar angle of $\vartheta = 90°$ and azimuthally in the horizontal polarization plane (blue) of the incident γ-ray beam or perpendicular to it (red). The luminosity profile of the $\vec{\gamma}$-ray beam in arbitrary units is indicated in gray by the dashed Gaussian curve. In the spectrum shown in panel (a) the 1_3^+ state is not excited. Hence, the peak observed at 3100 keV is the ground-state decay of hitherto unknown state(s).h Their transitions to the 2_1^+ state are located at 3027 keV [cf. panels (a) and (c)] and decays to the 2_γ^+ state are visible in the coincidence spectrum [panel (b)]. The spectra shown in panel (c) are dominated by the $1_i^+ \to 0_1^+$ ($i = 1$–3) transitions at 3111.0(4), 3159.1(4), and 3173.6(4) keV and the transitions to the 2_1^+ state at 3037.8(4), 3085.3(4), and 3100.1(4) keV, respectively. Already from these spectra the significant deviation from the Alaga prediction of 0.5 for the reduced relative decay-intensity ratios becomes evident for the 1_3^+ state (cf. Table I). The peak stemming from the $1_3^+ \to 2_\gamma^+$ transition is visible in the coincidence spectrum shown in panel (d).

contributions from excited states of other dysprosium isotopes contained in the target material [20] nor background radiation. The formation of a doublet of states at 3173 keV with different spin or parity quantum numbers is excluded due the beam's high energy resolution and the measured angular intensity asymmetries that agree uniquely with pure $M1$ transitions to the ground-state band. The measured $I_{1\to2/0}^{\text{rel}}$ values for the decay transitions to the ground-state band can be compared to parameter-free predictions from the Alaga rule [8]

$$R(\Delta K) = \frac{B(\pi\lambda; J_{K_i} \to J'_{K_f})}{B(\pi\lambda; \tilde{J}_{K_i} \to \tilde{J}'_{K_f})} = \left(\frac{C_{JK_i\lambda\Delta K}^{J'K_f}}{C_{\tilde{J}K_i\lambda\Delta K}^{\tilde{J}'K_f}}\right)^2, \quad (2)$$

where C are Clebsch-Gordan coefficients and J_{K_i}, J'_{K_f}, $\tilde{J}_{K_i}$, and $\tilde{J}'_{K_f}$ denote arbitrary states of two rotational bands differing by $\Delta K = K_f - K_i$. In the discussed case of $J^\pi = 1^+$ states, which predominantly decay to the 0_1^+ and 2_1^+ states of the ground-state band, Eq. (2) is reduced to

TABLE I. Comparison of reduced relative decay-intensity ratios, obtained from measured $I_{1\to2/0}^{\text{rel}}$ values, to predictions from the Alaga rule [8]. The transition to the 2_γ^+ state at 761.815(7) keV was directly observed only for the state at 3100 keV and the 1_3^+ state at 3173.6(3) keV [cf. Fig. 2, panels (b) and (d)]. The intensities are normalized to 100 for ground-state transitions. Absolute $M1$ strengths of ground-state transitions are taken from Refs. [16–18].

Transition	Energy (keV)	$B(M1; 1_i^+ \to J_f^+)$ (μ_N^2)	$R_{1\to f/0}^{\text{exp}}$ (%)	Alaga[a] (%)
$1_1^+ \to 0_1^+$	3111.0(4)	0.357(13)	100(7)	100
$1_1^+ \to 2_1^+$	3037.8(4)	$0.154^{+0.015}_{-0.016}$	43(4)	50
$1_1^+ \to 2_\gamma^+$	2349.4(4)		$\leq 21(8)$	
$1_2^+ \to 0_1^+$	3159.1(4)	0.317(12)	100(6)	100
$1_2^+ \to 2_1^+$	3085.3(4)	0.202(15)	64(4)	50
$1_2^+ \to 2_\gamma^+$	2397.3(4)		$\leq 8(3)$	
$1_3^+ \to 0_1^+$	3173.6(4)	0.273(11)	100(6)	100
$1_3^+ \to 2_1^+$	3100.1(4)	$0.079^{+0.006}_{-0.007}$	29(2)	50
$1_3^+ \to 2_\gamma^+$	2411.8(4)		16(3)	

[a]Assuming $K = 1$ for 1_i^+ ($i = 1$–3).

$$R(\Delta K) = \begin{cases} 2.0 & \text{for} \quad K_i = 0 \wedge K_f = 0 \quad \text{and} \\ 0.5 & \text{for} \quad K_i = 1 \wedge K_f = 0. \end{cases} \quad (3)$$

The measured decay branching ratios of known 1^+ states into the $K = 0$ ground-state band of predominantly axially deformed nuclei agree typically with the Alaga rule for $\Delta K = 1$ within experimental uncertainties [7,21–24]. The Alaga-rule expectations along with the reduced relative decay-intensity ratios $R_{1\to2/0}^{\text{exp}} = I_{1\to2/0}^{\text{rel}}(E_{\gamma,0}/E_{\gamma,2})^{2\lambda+1}$ [10,11] and $M1$ transition strengths into the ground-state band are summarized in Table I.

Discussion.—The 1^+ states near 3 MeV have previously [16,17] been identified as the main fragments of the scissors mode. The latter is characterized by the projection $K = 1$ [25–28] and, hence, is defined to decay into the ground-state band with a branching ratio of $R_{1\to2/0}^{\text{exp}} = 0.5$ according to the Alaga rule (3). However, significant deviations occur for the close-lying 1_2^+ and 1_3^+ states at 3159.1(3) and 3173.6(3) keV, respectively. For the latter, an 11σ violation of the Alaga rule is observed and needs to be clarified. Because of the overwhelming evidence for a quite pure $K = 0$ assignment to the 2_1^+ state of the ground-state rotational band of ^{164}Dy from many other decay branches, the violation of the Alaga rule must be caused by the structures of the involved 1^+ states. In order to cause such a substantial deviation for the reduced relative decay-intensity ratio into the $K = 0$ ground-state band, an additional contribution apart from a pure $K = 1$ component to the wave function of the 1_3^+ state is needed. This contribution can only be provided by a $K = 0$ component in the wave function. In this

situation, it is sufficient to consider a two-state mixing (TSM) scenario. The observed substantial deviation from the Alaga rule requires a mixing partner with comparable decay strength. The two close-lying states at 3159.1(3) and 3173.6(3) keV feature nearly equal $M1$ decay strengths. In fact, they belong to the strongest fragments of the scissors mode in the entire nuclear chart. Since both of them exhibit significant deviations from the Alaga rule, it is an obvious choice to consider them in the K-mixing scenario [21,29,30]. In a two-state K-mixing model, the wave functions of the 1_2^+ and 1_3^+ states can be written as linear combinations of basis states with pure $K=0$ and $K=1$ projections, i.e.,

$$|1_3{}^+, 3173 \text{ keV}\rangle = \alpha|1_{K=0}^+\rangle + \beta|1_{K=1}^+\rangle \quad \text{and}$$
$$|1_2{}^+, 3159 \text{ keV}\rangle = -\beta|1_{K=0}^+\rangle + \alpha|1_{K=1}^+\rangle, \quad (4)$$

where α and β are amplitudes normalized to $\alpha^2 + \beta^2 = 1$. Their ratio β/α is denoted γ and Z is the ratio of the doubly reduced [31] $\Delta K = 1$ and $\Delta K = 0$ $M1$ matrix elements. The mixing amplitudes are determined using the experimental intensity ratios (cf. Table I) and the ratio of transition strengths $B_{2/3} := B(M1; 0_1^+ \to 1_2^+)/B(M1; 0_1^+ \to 1_3^+)$ as input. Their transition widths were previously obtained in nuclear resonance fluorescence measurements [16–18]. As shown in Fig. 3, the data clearly favor a solution with comparably strong mixing and a
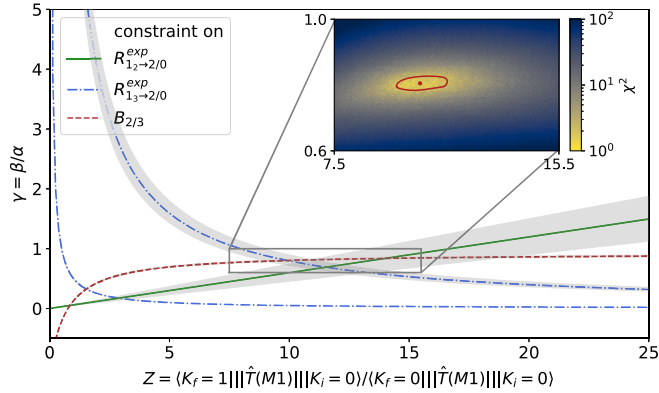

FIG. 3. Representation of experimental constraints in the $Z - \gamma$ plane of the TSM parameters. The solid green and blue dash-dotted lines indicate experimental constraints along with their gray uncertainty bands obtained from the measured relative decay-intensity ratios $R_{1_2 \to 2/0}^{\exp}$ and $R_{1_3 \to 2/0}^{\exp}$, respectively. The occurrence of two blue lines originates from an undefined phase in the initial problem. The intersections of these constraints describe potential solutions (Z, γ) to the TSM scenario. Additional information on the ratio of excitation strengths of the involved states $B_{2/3}$, is shown by a red dashed line. The inlay depicts the compatibility of parameter tuples (Z, γ) in terms of χ^2 values. The most probable solution, which corresponds to minimum χ^2, is indicated by a red dot along with its 1σ confidence interval. It exhibits a 46 times smaller χ^2 value than the solution with little mixing.

dominant $\Delta K = 1$ matrix element. The resulting parameters of the TSM are

$$\alpha^2 = 0.60_{-0.01}^{+0.02} \qquad \beta^2 = 0.40_{-0.02}^{+0.01}$$

$$Z = \frac{\langle K_f = 1|||\hat{T}(M1)|||K_i = 0\rangle}{\langle K_f = 0|||\hat{T}(M1)|||K_i = 0\rangle} = 10.6_{-0.7}^{+1.0}, \quad (5)$$

where $\hat{T}(M1)$ denotes the magnetic dipole transition operator. The mixing matrix element amounts to 6.85(4) keV corresponding to the unperturbed energies 3164.5(2) and 3167.5(2) keV. The two TSM parameters, γ and Z, along with an overall scale, enable a reproduction of all four involved $M1$ transition strengths as shown in Fig. 4. In addition, the TSM is consistent with the observed transitions to the 2_γ^+ state at 761.815(7) keV assuming $E2$ multipolarity. Because of the forbidden $\langle 2_\gamma^+||\hat{T}(M1)||1_{K=0}^+\rangle$ matrix element, any $M1$ transition from a $J = 1$ state to the $K = 2$ γ band must occur from the $K = 1$ component of the 1^+ state, implying a stronger transition to the 2_γ^+ state from the 1_2^+ state with 60% $K = 1$ component than from the 1_3^+ state, in contradiction to the data. Moreover, recent experiments [32,33] indicate that the $M1$ matrix element connecting $1_{K=1}^+$ and 2_γ^+ states is minuscule for deformed nuclei. Hence, the transition to the γ-vibrational band is tentatively attributed to the $K = 0$ component of the 1^+ states leaving electric quadrupole ($E2$) multipolarity as the only option. Indeed, while $M1$ character cannot be excluded, $E2$ radiation is consistent with the observed angular intensity distribution. Assuming pure $E2$ character, the associated transition strength $B(E2; 1_3^+ \to 2_\gamma^+) = 2.05_{-0.42}^{+0.35}$ W.u. is comparable
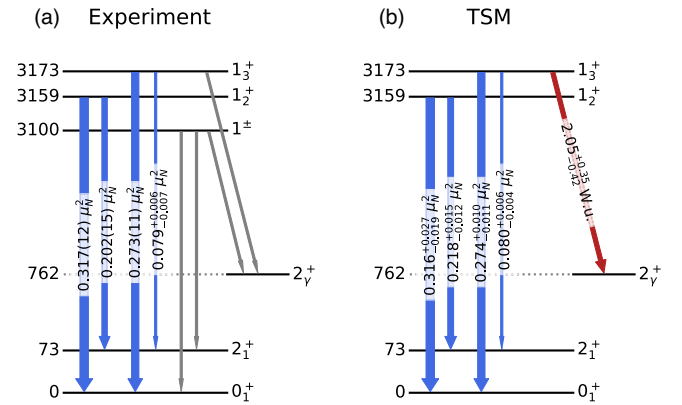

FIG. 4. Comparison of level schemes obtained from experimental data [panel (a)] and the TSM analysis [panel (b)]. Absolute transition strengths $B(M1; 1_{2,3}^+ \to 0_1^+)$ are taken from Ref. [18]. All other transition strengths were calculated from the decay-intensity ratios which were determined in the present work. Based on the given uncertainties, the agreement is excellent for each individual transition. The $E2$ transition between the 1_3^+ and 2_γ^+ states is shown in red. This assignment was possible only due to the identification of a prevailing $K = 0$ component in the wave function of the 1_3^+ state.

to the $\Delta K = 2$ $E2$ decay strength of the γ band to the ground-state band of ^{164}Dy [34]. For the 1_2^+ state, an upper limit for the decay to the 2_γ^+ state was established (cf. Table I). From the $\Delta K = 0$ matrix element, which is obtained from the TSM analysis, the excitation strength of the unperturbed $1_{K=0}^+$ basis state is determined to

$$B(M1; 0_1^+ \to 1_{K=0}^+) = 0.008(1)\mu_N^2. \qquad (6)$$

This value represents the first experimental extraction of this property. It agrees within a factor of four with predictions [35] from the quasiparticle phonon nuclear model [36] that revealed only a small number of the $1_{K=0}^+$ states of ^{164}Dy with excitation energies around 3 MeV. These are two-quasiparticle neutron states with $B(M1; 0_1^+ \to 1_{K=0}^+)$ values equal to 0.034, 0.037, and 0.002 μ_N^2. Based on typical $M1$ excitation strengths of this size it is plausible that pure $1_{K=0}^+$ states hitherto escaped detection. Merely a high-precision measurement of experimental decay-intensity ratios—as presented here—along with additional information on the excitation strengths of the involved states enables the disentanglement of $K = 0$ and $K = 1$ components.

Future systematic studies of $1_{K=0}^+$ states might profit from the previous observations on the decay behavior of $J^\pi = 1^+$ scissors mode states. The extensive compilation of data [22,24] on the experimental decay-intensity ratio $I_{1\to2/0}^{\rm rel}$ [7,21] might indicate the widespread existence of 1^+ states with a decay behavior deviating from the Alaga predictions (3). This can serve as a starting point for systematic measurements of $B(M1; \Delta K = 0)$ excitation strengths of axially deformed even-even nuclei and an exploration of positive-parity states with negative $\mathcal{R}$ symmetry.

Summary.—For the first time experimental information on the excitation strength of a $1_{K=0}^+$ state has been presented. This state can be interpreted in terms of a negative $\mathcal{R}$-symmetry quantum number. Employing a two-state model analysis with experimental intensity ratios and $M1$ excitation strengths as input, the $B(M1; 0_1^+ \to 1_{K=0}^+)$ value has been determined. The result is in semiquantitative agreement with calculations in the quasiparticle-phonon nuclear model. It provides first quantitative evidence for the small size of $\Delta K = 0$ $M1$ excitation strengths of axially deformed even-even nuclei as compared to $\Delta K = 1$ transitions. The observed γ decay of the 1_3^+ state of ^{164}Dy to the γ band was attributed to the $K = 0$ component of its wave function. Experimental establishment of its predominant $E2$ character is very important for further insight into nuclear states with negative $\mathcal{R}$ quantum number.

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